Examination of an Analogy Toward the Understanding of Thermal Lens Oscillations

G. Gouesbet* and J. Maquet†
Laboratoire d'Energétique des Systèmes et Procédés, Mont-Saint-Aignan, France

The overstability for surface tension and coupled buoyancy-driven instability in a horizontal liquid layer, with very general conditions, is studied. A linear formulation to compute the critical quantities is established. Numerical results are given and compared with experiments in which a free surface is heated by a controlled hot-wire located near and below it. When correctly presented in terms of well-chosen reduced quantities, theoretical and experimental results agree very well, showing that there is an analogy between the theoretical problem (horizontal liquid layer, basic conductive state) and the experimental situation (hot-wire heating, basic convective state). Disagreements are pointed out to stress the limitations of the analogy. The original motivation of the work is the understanding of thermal lens oscillations produced when heating below the free surface is carried out using a laser beam.

Nomenclature

Bo	= Bond number
Cri	= Crispation number
d_{hw}	= distance between the hot-wire and the free surface (HWE) or between solid wall and free surface (model)
D	= operator d/dz
f	= planform function
g	= gravity acceleration (buoyancy), >0 when the upside of the layer is a free surface
g_s	= gravity acceleration (gravity waves)
H	= mean curvature of the free surface or heat transfer coefficient at the free surface
k_T	= adverse temperature gradient ($-dT/dz$)
K_T,$K_{T,\mathrm{eff}}$	= thermal and efficient thermal diffusivities
Ma	= Marangoni number
$\boldsymbol{n}_i$	= unit vector
Nu	= Nusselt number (heat transfer at the free surface)
p_i	= pressure in the medium i
p	= pressure perturbation
Pr	= Prandtl number
Ra	= Rayleigh number
t	= time
T	= liquid temperature
T_0	= reference temperature
T_s	= free surface temperature in the nonperturbed state
U_i	= liquid velocity
u_i	= liquid velocity perturbation
$u_{fs,i}$	= free surface velocity perturbation
Vi	= viscosity number
x,y,z	= Cartesian coordinate system
$z_{f,s}$	= free surface location in the basic state
Greek symbols	
α	= wave number of the normal mode
α_T	= expansion factor
β	= time constant
Γ	= rate of variation of the surface tension with temperature
Δ_{II}	= surface Laplacian operator
δ_s	= z dependent part of the perturbation of the free surface location
δT_*	= critical temperature difference (theory)
ΔT_*	= critical temperature difference (experiment)
δz_{fs}	= free surface perturbation
ζ	= z dependent part of the velocity perturbation
θ	= z dependent part of the temperature perturbation
$\kappa+\varepsilon$	= sum of the interfacial viscosities
μ	= dynamic viscosity
ρ	= fluid density
ρ_0	= liquid density at the reference temperature T_0
σ	= surface tension
τ	= temperature perturbation
υ	= z dependent part of the pressure perturbation
χ_{eff}	= efficiency factor for thermal diffusivities
$\bar{\omega}$	= reduced time constant

Introduction

THE aim of the paper is to approach the understanding of the instability mechanisms acting to produce thermal lens oscillations when a CW laser beam travels horizontally near and below a free surface, in an absorbing liquid.[1] The oscillatory character of the system is directly evidenced by projecting the outgoing beam onto a screen. These oscillatory optical phenomena are accompanied by oscillatory motion of the free surface and convective oscillations in the bulk.

To understand these effects, a simpler model experiment was designed in which heating below the free surface is carried out by means of a temperature-controlled hot-wire (HWE: hot-wire experiments). Increasing the hot-wire temperature leads to a supercritical oscillatory instability. Critical temperature differences (between hot-wire and ambient temperature far above the free surface) and critical frequencies have been measured. Experiments were performed with a series of silicone oils having well-known thermophysical properties.[1,2]

A theory able to predict the critical quantities measured in the HWE would be a significant step toward the understanding of the thermal lens oscillations. A numerical rigorous approach is currently developed.[3] In the present paper, we shall conversely examine whether the problem of overstability in a horizontal liquid layer (with a basic conductive state), under surface tension and buoyancy agencies, presents some quantitative analogy with the HWE results (with a basic convective state). If the answer is positive, we shall examine to which extent the analogy holds. To model the upward convective heat transfer to the free surface, present in the HWE but lacking in the model problem, we shall replace the actual thermal diffusivity of the liquid K_T by an efficient thermal

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*Professor. Member AIAA.
†Professor.

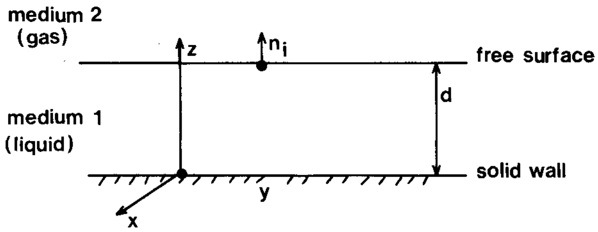

Fig. 1 Geometry of the problem.

diffusivity $K_{T,\text{eff}}$, according to the relation

$$K_{T,\text{eff}} = K_T(1+\chi_{\text{eff}}), \qquad \chi_{\text{eff}} \geq 0 \tag{1}$$

where χ_{eff} is an efficiency factor.

Independently of our original motivation, the present paper completes to some extent a tradition concerning the study of Rayleigh-Bénard-Marangoni instabilities. It can be considered as a generalization of Takashima's work, which is devoted to a pure surface tension (Marangoni) mechanism.[4]

Equations at the Free Surface

The liquid layer of infinite extent is limited by a solid wall and a free surface (Fig. 1). The positive z is directed toward the free surface. The Cartesian coordinate system (x,y,z) is composed of two axes x and y located at the solid wall. The free surface location z_{fs} is given by

$$z_{fs} = d_{hw} + \delta z_{fs} \tag{2}$$

where d_{hw} is the nonperturbed location and δz_{fs} the location perturbation.

The kinematic condition for the free surface reads

$$\boldsymbol{n}_i u_{fs,i} = u_{fs,z} = \frac{\partial z_{fs}}{\partial t} \tag{3}$$

where $u_{fs,i}$ is the free surface velocity for a perturbed state.

The free surface is assumed to be a two-dimensional-Newtonian fluid, nearly plane, with a negligible mass. We neglect the viscous force from the gaseous medium and find that the surface dynamics satisfies[5,6,7]:

1) A normal force balance

$$2H\sigma = \rho g_s \delta z_{fs} - (p_1 - p_2) + 2\mu \frac{\partial U_3}{\partial z}, \qquad z = z_{fs} \tag{4}$$

2) A tangential force balance

$$\mu \Delta_{\text{II}} U_3 - \mu \frac{\partial^2 U_3}{\partial z^2} = \Delta_{\text{II}}\sigma - (\kappa + \varepsilon)\Delta_{\text{II}} \frac{\partial U_3}{\partial z} \tag{5}$$

in which H is the surface mean curvature equal to $(\frac{1}{2})\Delta_{\text{II}} z_{fs}$, Δ_{II} is the surface Laplacian $(\partial^2/\partial x^2 + \partial^2/\partial y^2)$, σ the surface tension, ρ and μ the specific mass and the dynamic viscosity (constant) of the liquid, p_i the pressure in the medium i, U_3 the vertical velocity in the bulk, and $(\kappa+\varepsilon)$ the sum of the interfacial viscosities also assumed to be constant. Finally, g_s is actually equal to the value of g used for the bulk buoyancy term (next section). A positive g_s (or g) means that the upside of the layer is a free surface. The subscript s stands for "surface" and is used to keep track of the corresponding term (gravity waves) throughout the formulation and computations.

Bulk Equations

We assume that the liquid complies with the Oberbeck-Boussinesq approximation.[8,9] Then, for the nonperturbed (basic) state, we write the continuity, momentum, and the temperature equations. To produce the linear equations, we call u_i, τ, p the perturbations for the velocity U_i, the temperature T, and the pressure p_i. We write the continuity, momentum, and temperature equations for the perturbed state. Subtracting, neglecting the terms that are nonlinear with respect to the perturbations, and assuming that the basic state is a conductive (motionless) one, we obtain the continuity, the momentum, and the temperature equations for the perturbations. Then we see that the space and time are separable in the perturbations, and decompose the spatial dependence in terms of normal modes.

$$(p,u_i,\tau) = e^{\beta t} f(x,y)[\upsilon(z),\zeta(z),\theta(z)] \tag{6}$$

in which the planform function $f(x,y)$ satisfies the Helmholtz equation

$$\Delta_{\text{II}} f + \alpha^2 f = 0 \tag{7}$$

in which α is the wave-number of the normal mode. Assuming $\partial T/\partial x_j = (0,0,-k_T)$, we obtain, after fairly lengthy but rather standard computations, a velocity equation, and a temperature equation

$$(D^2 - \alpha^2 r^2)(D^2 - \alpha^2)\zeta = (\rho_0/\mu)\alpha_T g \alpha^2 \theta \tag{8}$$

$$\mu K_{T,\text{eff}}(D^2 - \alpha^2 q_{\text{eff}}^2)(D^2 - \alpha^2 r^2)(D^2 - \alpha^2)\theta = -k_T \rho_0 \alpha_T g \alpha^2 \theta \tag{9}$$

in which

$$r^2 = 1 + \frac{\rho_0 \beta}{\mu \alpha^2}, \qquad q_{\text{eff}}^2 = 1 + \frac{\beta}{K_{T,\text{eff}}\alpha^2} \tag{10}$$

We also obtain a pressure equation

$$(D^2 - \alpha^2)\upsilon = \rho_0 \alpha_T g D\theta \tag{11}$$

which can be modified using a momentum equation, leading to

$$\alpha^2 \upsilon = \mu(D^2 - \alpha^2 r^2)D\zeta \tag{12}$$

Also, from a temperature equation, we relate the temperature and velocity z-perturbations

$$(D^2 - \alpha^2 q_{\text{eff}}^2)\theta = -(k_T/K_{T,\text{eff}})\zeta \tag{13}$$

Three-Constant Solution for θ

The θ-solution of Eq. (9) has been researched under the form

$$\theta = \sum_{k=0}^{\infty} c_k z^k \tag{14}$$

subject to a double kinematic condition at the solid surface ($\zeta = D\zeta = 0$) and to a thermal condition chosen to be $\theta = 0$ (conductor case). The alternative condition ($D\theta = 0$, insulating case) is not presented for lack of space.

We find that θ is the sum of an odd and even series, with three arbitrary constants c_1, c_4, and c_5. The result can be expressed by the following relations:

$$c_{2k} = -\frac{4!}{(2k)!} E_4^{2k} c_4, \qquad k \geq 0 \tag{15}$$

$$c_{2k+1} = -\frac{5!}{(2k+1)!} E_5^{2k+1} c_5 - \frac{1!}{(2k+1)!} E_1^{2k+1} c_1, \qquad k \geq 0 \tag{16}$$

with

$$E_4^0 = E_4^2 = E_5^1 = E_5^3 = E_1^5 = 0 \tag{17}$$

$$E_4^4 = E_1^1 = E_5^5 = -1 \quad (18)$$

$$E_1^3 = -\alpha^2 q_{\mathrm{eff}}^2 \quad (19)$$

$$(E_4^{2k+6}, E_i^{2k+7}) = -A_4(E_4^{2k+4}, E_i^{2k+5}) - A_2(E_4^{2k+2}, E_i^{2k+3}) - A_0(E_4^{2k}, E_i^{2k+1}), \quad i = (1,5), \quad k \geq 0 \quad (20)$$

in which

$$A_4 = -\alpha^2(1 + r^2 + q_{\mathrm{eff}}^2) \quad (21)$$

$$A_2 = \alpha^4(r^2 + r^2 q_{\mathrm{eff}}^2 + q_{\mathrm{eff}}^2) \quad (22)$$

$$A_0 = \left(\frac{Ra}{(1+\chi_{\mathrm{eff}})d_{hw}^4} - \alpha^4 q_{\mathrm{eff}}^2 r^2\right)\alpha^2 \quad (23)$$

where Ra is the Rayleigh number

$$Ra = \frac{g\alpha_T k_T d_{hw}^4 \rho_0}{\mu K_T} \quad (24)$$

Free Surface Boundary Conditions

The location of the free surface is not determined for the perturbed state. Consequently, we have four coefficients to determine: the three above (c_1, c_4, c_5) for θ, plus a fourth one for the free surface location. This requires four boundary conditions at the free surface.

First Condition

The perturbation δz_{fs} is written [as is Eq. (6)]

$$\delta z_{fs} = e^{\beta t} f \delta_s(z) \quad (25)$$

with a similar expression for $u_{fs,z}$. The kinematic condition [Eq. (3)] becomes

$$\zeta = \beta \delta_s, \qquad z = z_{fs} \quad (26)$$

Then, we invoke Eq. (13) and obtain the first condition

$$\frac{1}{\alpha^2}(D^2 - \alpha^2 q_{\mathrm{eff}}^2)\theta + k_T \omega \frac{Pr}{1+\chi_{\mathrm{eff}}}\delta_s = 0, \qquad z = z_{fs} \quad (27)$$

in which $\bar{\omega}$ is the reduced time constant and Pr the Prandtl number

$$\omega = \frac{\beta}{v\alpha^2}, \qquad Pr = \frac{\mu}{\rho_0 K_T} \quad (28)$$

Second Condition

We start from the normal force balance [Eq. (4)], give to $U_3 = u_{fs,z}$ its expression obtained from Eq. (6), and similarly for δz_{fs}, which appears also in the computation of the mean curvature H. We write $\sigma = \sigma_s$ (neglecting higher order terms) and recognize that $(p_1 - p_2) = p$, the pressure perturbation. We also use Eqs. (12) and (13) to which we apply the operator D. We obtain the second boundary condition

$$Cri(1+\chi_{\mathrm{eff}})d_{hw}^5(D^2 - \alpha^2(r^2+2))(D^2 - \alpha^2 q_{\mathrm{eff}}^2)D\theta + (\alpha^4 d_{hw}^4)(\delta_s k_T) + Bo(\alpha^2 d_{hw}^2)(\delta_s k_T) = 0, \qquad z = z_{fs} \quad (29)$$

where Cri and Bo are the crispation and Bond numbers

$$Cri = \frac{\mu K_T}{\sigma_s d_{hw}}, \qquad Bo = \frac{\rho_0 g_s d_{hw}^2}{\sigma_s} \quad (30)$$

Third Condition

We start from the tangential force balance [Eq. (5)]. We express the perturbations according to Eq. (6). We write the surface tension $\sigma(z_{fs})$ in the perturbed state as follows:

$$\sigma(z_{fs}) = \sigma(T_s) + \Gamma[\tilde{T}(z_{fs}) - T_s] \quad (31)$$

in which

$$\Gamma = \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_s} \quad (32)$$

in which T_s is the free surface temperature in the basic state, and $\tilde{T}(z_{fs})$ is the temperature of the free surface at the perturbed location z_{fs}

$$\tilde{T}(z_{fs}) = T(z_{fs}) + \tau(z_{fs}) \quad (33)$$

where $T(z_{fs})$ is not equal to T_s because it is the temperature in the absence of any perturbation but at the perturbed location. From a Taylor expansion

$$T(z_{fs}) = T_s - k_T \delta z_{fs} \quad (34)$$

whence

$$\tilde{T}(z_{fs}) = T_s - k_T \delta z_{fs} + \tau(z_{fs}) \quad (35)$$

Injecting Eq. (35) in Eq. (31), and applying the surface Laplacian operator, we obtain

$$\Delta_{\mathrm{II}}\sigma = \Gamma\{\Delta_{\mathrm{II}}\tau - k_T \Delta_{\mathrm{II}}\delta z_{fs}\}, \qquad z = z_{fs} \quad (36)$$

Equation (36) is injected in the tangential force balance. Furthermore, we use the Helmholtz equation for f and invoke Eqs. (13) and (26) to obtain the third condition at the free surface

$$d_{hw}^4(D^2+\alpha^2)(D^2 - \alpha^2 q_{\mathrm{eff}}^2)\theta + Vi(\alpha^2 d_{hw}^2)d_{hw}^3(D^2 - \alpha^2 q_{\mathrm{eff}}^2)D\theta - \frac{Ma}{1+\chi_{\mathrm{eff}}}(\alpha^2 d_{hw}^2)\theta + \frac{Ma}{1+\chi_{\mathrm{eff}}}(\alpha^2 d_{hw}^2)k_T\delta_s = 0, \qquad z = z_{fs} \quad (37)$$

in which Vi and Ma are the viscosity and Marangoni numbers

$$Vi = \frac{\kappa + \varepsilon}{\mu d_{hw}}, \qquad Ma = \frac{-\Gamma k_T d_{hw}^2}{\mu K_T} \quad (38)$$

Fourth Condition

The fourth condition at the free surface is obtained by invoking the Newton heat transfer law, in the basic and perturbed state, respectively, subtracting and rearranging, lead to

$$d_{hw}D\theta + Nu\theta - Nu k_T \delta_s = 0, \qquad z = z_{fs} \quad (39)$$

Characteristic Equation and the Problem Solution

We introduce the free surface boundary conditions [Eqs. (27), (29), (37), and (39)] into the three-constant solution for θ. We obtain a linear homogeneous set having the form $A_{ij}V_j = 0$, in which V_j is the unknown vector $(c_1, c_4, c_5, k_T\delta_s)$. For the occurrence of nontrivial solutions ($\neq 0$), the characteristic determinant of the set must be equal to 0. This condition leads to

$$Ma_{\mathrm{eff}} = \frac{Ma}{1+\chi_{\mathrm{eff}}} = \frac{N}{D} \quad (40)$$

with

$$N = \begin{vmatrix} R_1 & R_4 & R_5 & \dfrac{\bar{\omega}Pr}{1+\chi_{\mathrm{eff}}} \\ S_1 & S_4 & S_5 & \dfrac{-(Bo+\alpha^2 d_{hw}^2)}{Cri(1+\chi_{\mathrm{eff}})} \\ (T_1 + U_1 Vi) & (T_4 + U_4 Vi) & (T_5 + U_5 Vi) & 0 \\ (X_1 + Nu Y_1) & (X_4 + Nu Y_4) & (X_5 + Nu Y_5) & Nu/(\alpha^2 d_{hw}^2) \end{vmatrix} \quad (41)$$

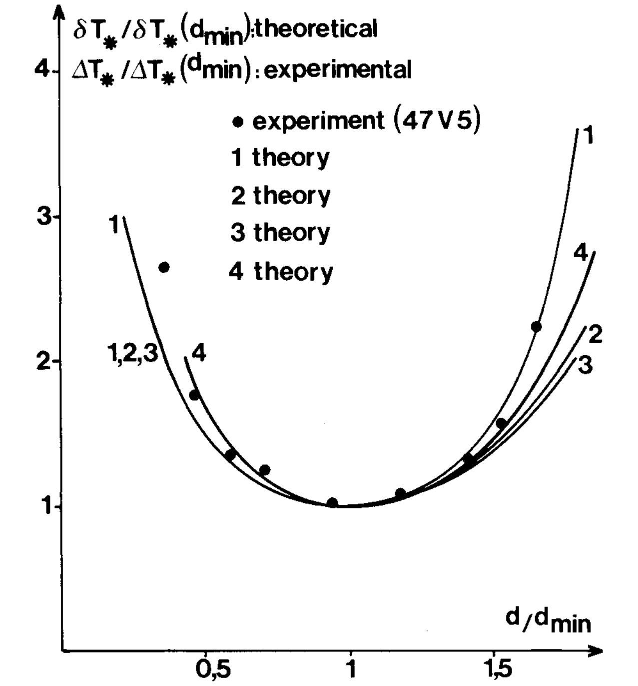

Fig. 2 Reduced critical temperature difference vs wire-surface distance (labels 1–4).

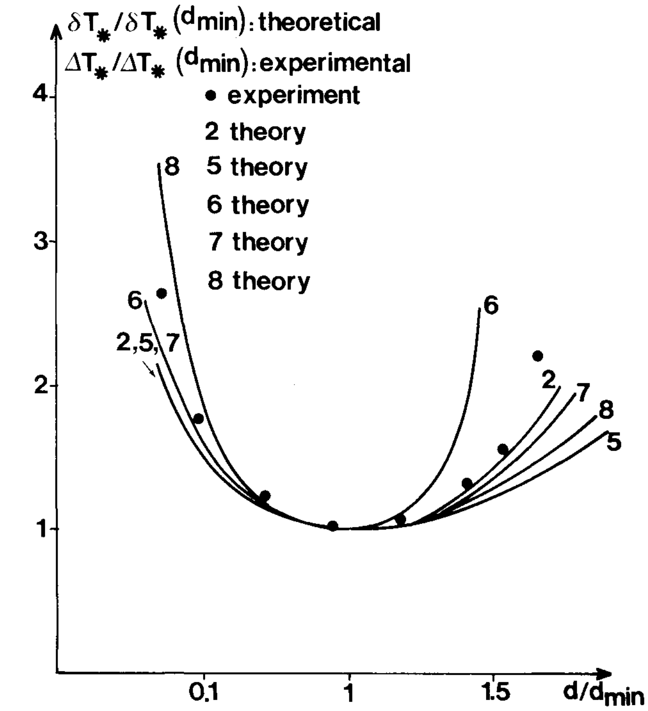

Fig. 3 Reduced critical temperature difference vs wire-surface distance (labels 5–8).

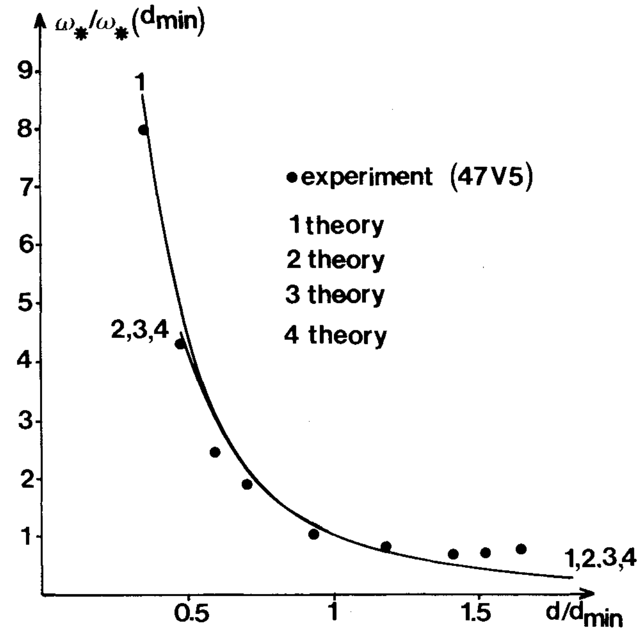

Fig. 4 Reduced critical frequency vs wire-surface distance (labels 1–4).

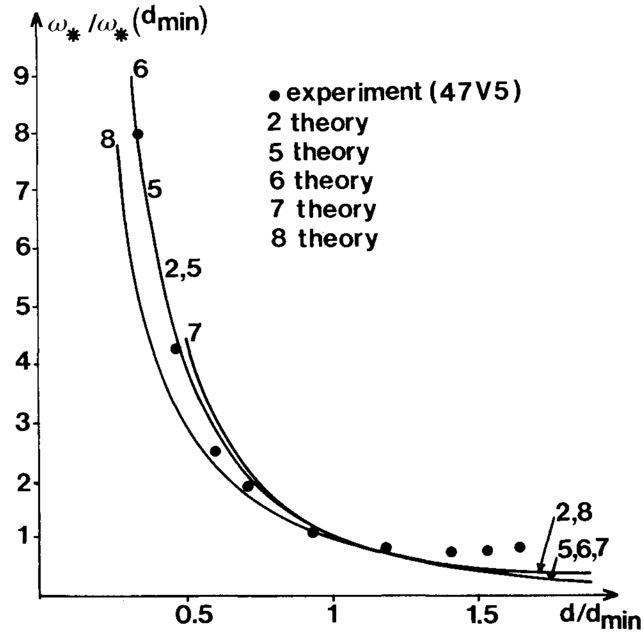

Fig. 5 Reduced critical frequency vs wire-surface distance (labels 5–8).

$$D = \begin{vmatrix} R_1 & R_4 & R_5 & -\bar{\omega}\dfrac{Pr}{1+\chi_{\mathrm{eff}}} \\ S_1 & S_4 & S_5 & \dfrac{(Bo+\alpha^2 d_{hw}^2)}{Cri(1+\chi_{\mathrm{eff}})} \\ V_1 & V_4 & V_5 & 1 \\ (X_1+NuY_1) & (X_4+NuY_4) & (X_5+NuY_5) & -Nu/(\alpha^2 d_{hw}^2) \end{vmatrix} \tag{42}$$

in which the quantities $R_1, R_2, \ldots, Y_5$ depend on $\alpha, d_{hw}, q_{\mathrm{eff}}, r$, and on the E_i^j's. These expressions are not given here to avoid overloading this paper, but they are available on request (or can be readily derived).

For the marginal state of overstability we are looking for

$$\beta = i\omega \tag{43}$$

The characteristic Eq. (40) is split into two equations that must be satisfied simultaneously. The first one (label A) states that Ma_{eff} is a real number. The second one (label B) states that Ma_{eff} must be equal to $Ma/(1+\chi_{\mathrm{eff}})$, in which Ma is given by the relation (38).

In general, the two equations are coupled leading to a difficult numerical work to solve them. For a given wavenumber α, the solution, if it exists, takes the form of a sequence of overstable imaginary time constants β_i, the β-spectrum, associated to a sequence of critical adverse temperature gradients $k_{T,i}$, the k_T-spectrum. The critical values for a given α, namely β_c (or ω_c) and $k_{T,c}$, correspond, among the spectra values, to the subscript i, for which $|k_{T,i}|$ is the smallest. These values are associated with a critical Marangoni number Ma_c.

Scanning over α, we can determine the functions, $\beta_c(\alpha)$, $k_{T,c}(\alpha)$, and $Ma_c(\alpha)$. The α-value, for which $|k_{T,c}(\alpha)|$ is minimum, is the critical wave-number α_* associated with the final critical quantities β_*, k_{T*}, and Ma_*.

Numerical computations are carried out using a computer program called HOTWI. From HOTWI, we derived another program, called TAKAS, adapted to a fully dimensionless formulation, which is not given in the present paper. Using TAKAS, we were able to reproduce perfectly well the results for overstability given by Takashima[4] for the special cases $Nu = Vi = Ra = 0$. This perfect agreement provides us with a means to check the correctness of our formulation and the accuracy of the computer program HOTWI from which TAKAS derives.

Comparisons with the HW-Experiments

Experiments we intend to model (the HWE) are briefly discussed in the introduction and are more fully discussed in Refs. 1 and 2. Experimental results concerned critical temperature differences ΔT_* (between hot-wire and ambient air) and critical frequencies f_*(Hz), equal to $\beta_*/(2\pi i)$, vs the wire-surface distance d_{hw}. They were obtained for four Rhodorsil silicon oils labelled 47V5, 47V10, 47V50, and 47V100. The present comparisons will concern the 47V5 oil. (Results would be similar for the other oils.) The thermophysical properties of the 47V5 oil are, in SI-units at 25°C: $\mu/\rho_0 = 5 \cdot 10^{-6}$, $\rho_0 = 910$; $K_T = 6.69 \cdot 10^{-8}$, $\sigma = 19.7 \cdot 10^{-3}$, $\alpha_T = 1.05 \cdot 10^{-3}$, and $\Gamma = -7 \cdot 10^{-5}$.

Experimentally, the function $\Delta T_*(d_{hw})$ presents a minimum for $d_{hw} = d_{\min}$, with $d_{\min} = 0.085$ cm and $\Delta T_*(d_{\min}) = 29.5$ K. Figure 2 shows the reduced critical temperature differences $\Delta T_*/\Delta T_*(d_{\min})$ vs the reduced wire-surface distance $d_{hw}/d_{\min}$. These results are also reproduced in Fig. 3. The experimental critical frequencies are presented in Fig. 4 (and reproduced in Fig. 5) as $\omega_*/\omega_*(d_{\min})$ vs $d_{hw}/d_{\min}$.

In these figures, the theoretical predictions are reported. The theoretical profiles were labelled from 1 to 8. These labels correspond to the following conditions:

Label 1: Liquid layer suspended from a cold ceiling; conductor case; thermophysical and physical properties corresponding to the actual HWE, except for the efficiency factor χ_{eff} equal to 0.002; also, $(\kappa + \varepsilon) = Nu = 0$.

Label 2: Same as label 1, but with $\chi_{\text{eff}} = 16.94$.

Label 3: Same as label 2, but $Nu = 0.05$.

Label 4: Same as label 2, but $(\kappa + \epsilon) = 10^{-6}$ (SI-units).

Label 5: Same as label 2, but $g = 0$ (no buoyancy): pure surface-tension mechanism.

Label 6: Same as label 2, but $g_s = 0$ (no gravity waves).

Label 7: Same as label 2, but for an insulating thermal condition at the rigid wall. (The corresponding formulation is not given in the present paper for lack of space, but has been established.)

Label 8: Same as label 2, but for a liquid layer above a cold floor.

The discussion of the results is as follows:

For a hot rigid wall (liquid layer suspended from a hot ceiling or liquid layer above a hot floor), overstability is not observed in the computations. Overstability is only observed for a liquid layer suspended from a cold ceiling (labels 1–7) or for a liquid layer above a cold floor (label 8). For $d > d_{hw}$, the influence of the buoyancy and of the gravity waves mechanism is very important (labels 5 and 6). The buoyancy mechanism is stabilizing for overstability (even if it is destabilizing for an exchange of stability).

The original question as to whether there is a quantitative analogy between the HWE-experiments and the infinite layer problem can be answered positively. Let us dismiss the curves labelled 5 (no buoyancy) and 6 (no gravity waves), since buoyancy and gravity waves do exist in the HWE. Hence, all the theoretical profiles are very similar. They own a robust character with modifications of the input values. This robust character also appears in experiments, because the experimental results obtained for the four oils collapse into single curves when they are given with the same presentation as in Figs. 2–5.[10] The agreement between the theoretical reduced profiles and the experimental results is impressively good and defines the limits in which the quantitative analogy holds. Such a high agreement was not expected. We should have been content with similar trends. The good quality of the theoretical/experimental comparison is probably linked to the robust character of the theoretical results. Furthermore, we interpret it as evidence of the correct identifications of two instability mechanisms (buoyancy and Marangoni effects) in the HWE, and in the thermal lens oscillations.

However, the differences between theoretical and experimental nonreduced values remain important. The critical temperature difference is positive in the HWE, and negative in the computations. It corresponds to the fact that no overstability is theoretically observed for a hot rigid floor. But if, at first sight, the case of a liquid layer above a hot floor could seem more akin to the HWE-experiments, a more subtle discussion could lead to a different conclusion, for, in the HWE, the free surface is heated just above the hot-wire, whereas it is relatively colder far from the hot-wire location because of the liquid convection that brings a cold liquid up from the bottom of the cell.

The value of d_{hw} in the computations ranges from 0.044 cm (label 1) to 0.103 cm (label 8) and has the same order of magnitude as in the HWE (0.085 cm). But the absolute values of the critical temperature differences are much too high. They range from 426 K (label 8) to 5000 K (label 1), compared with 29.5 K for the experiments. Such a high temperature difference in the computations would even conflict with the very existence of the liquid state. Increase of χ_{eff} from the label 1 case to label 2 case helps to decrease $|\delta T_*|$ from 5000 to $\simeq 1560$ K. As expected, the efficiency factor helps to lower the critical temperature differences, but not enough to approach the experimental value because, when χ_{eff} increases above 16.94 (label 2), $|\delta T_*|$ does not decrease any more. For the critical frequencies, there is a difference of one order of magnitude between theory and experiments: ω_* ranges from 48 s^{-1} (label 6) to 167 s^{-1} (label 1), compared with 8.92 s^{-1} in the HWE.

Consequently, although two instability mechanisms (buoyancy, surface tension) have probably been correctly identified in the HWE-experiments (in so far as the observed agreements are not attributed to an artifact) the description is far from being complete. What is lacking is certainly linked to the convection present in the HWE-basic state, which is not a quiescent state, or to the fact that horizontal components of grad(T) exist in these experiments. Conversely, in the Fig. 1 problem, the basic state is motionless and the temperature gradient is vertical. Current efforts are devoted to the production of new experimental results and observations, and to more complete theoretical approaches in order to attempt to fill the gap still existing between experiments and the present state of the theoretical work. Besides rigorous numerical approaches developed, we are also examining an analogy with the same problem as in Fig. 1, to which we added a pure shear. We also plan to examine the relation with hydrothermal instabilities.[11,12] Finally, a very simple semiquantitative model, based on the existence of two main time scales (one associated with heat transfer from the wire to the free surface, and the other with the Marangoni effect) has been proposed.[13]

Also, the present paper is interesting in itself, independent of its reference to the hot-wire experiments. The study of overstability using a dimensionless formulation in the case of Fig. 1 will appear elsewhere as a sequel of the present work.[14]

Conclusion

The aim of the present work was to examine a possible analogy between the problem of onset of overstability in an infinite layer through buoyancy and surface tension mechanisms, and experimental overstability observed when a hot-wire is located near and below a free surface (HWE). The HWE are considered as an experimental model of a still more complicated phenomenon, namely, thermal lens oscillations produced when a laser beam propagates in an absorbing liquid medium, near and below the interface liquid/gas.

The theoretical infinite layer problem was examined using a linear analysis. The formulation to compute critical temperature differences, frequencies and wave-numbers is presented. Numerical results are compared with the HWE results. If the results are presented under appropriately reduced forms, the agreement between theory and experiments is very good indeed. However, in terms of nonreduced results, the disagreement between the theoretical model and the HWE is very high for the critical temperature differences and frequencies. This disagreement is attributed to the fact that the convection is not included in the basic state of the model. The obtained results nevertheless show that a significant step has been achieved towards the understanding of thermal lens oscillations.

If we do not refer to the HWE, the theoretical work presented in the paper is self-consistent. It is a contribution to the understanding of overstability through coupled buoyancy and surface-tension mechanisms, with general conditions at the free surface. Systematic results in this restricted framework will be produced later as a sequel to this work.

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